Limited water resource management using mathematical programming (case study: Arasbaran district)

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Abstract

In the present paper the use of mathematical programming theory is proposed to define optimum allocation of variable limited water resources in Arasbaran district (north-western Iran). Relationships between crop yield and applied water was simulated by Budget model by using multiplicative formula for various crops from year to year which depend on the deterministic component of the process of water exchange soil-crop-atmosphere. Net benefit of each crop yield determined by Mannochi and Mecarelli function and the values necessary for solving the expression were supplied by Moghan Agricultural research center. Cumulative profits (10 years) related to all possible combinations of crop pattern and of water supply using a mathematical optimization approach with linear programming by constrains that defined for farms. Finally finding the optimum solution for planning an optimal crop patterns for this areas and compared the net benefits as function of annual variable net available water volume (NIV=8, 10, 12, 14 and 16 MCM) for patterns.

Keywords: Arasbaran, Budget model, Limited water resources, linear programming

1. Introduction

When water is not limited or there is no constraint on irrigation water supplies, irrigation planning involves the optimal allocation of land to different crops under consideration to maximize the net returns from the scheme with irrigation supplies that satisfy maximum crop water requirements. But when water is limited, the allocation process is not only limited by area but also by available water. When water is limited there is always the possibility of some area being left with no irrigation, if irrigation is applied to meet maximum crop water requirements. When the crop is irrigated with this full irrigation depth, the last few increments of water applied to the crop result only in a small yield increment. If these last few increments of water are applied to some additional area, the total yields or net returns obtained from the scheme may be more (English and Nuss 1982; Trimmer 1990), though the yield per unit area is reduced. Thus, in a water limiting condition, the additional problem is to decide upon the last few increments for each crop compared with the additional area that can be irrigated by those increments so that the total net returns can be maximized. Therefore, it is necessary to consider a range of depths of irrigation water to be applied to each crop in the process of allocation (Gorantiwar and Smout 2005).

Optimization models have been used extensively in water resources systems analysis and planning (Loucks *et al*. 1981). Benli and Kodal (2003) developed a non-linear model for determination of the optimum cropping pattern, water allocation and farm income under inadequate and limited water supply conditions. The objective function of this model was developed based on crop water–benefit functions. A number of researchers have addressed the problem of allocation of a limited water supply for irrigation in a multicrop environment (Rao *et al*. 1990; Sunantara and Ramirez 1997; Paul *et al*. 2000; Reca *et al*.2001; Teixeira and Marino 2002; Umamahesh and Raju 2002; Gorantiwar and Smout 2003; Smout and Gorantiwar 2005).

The purpose of this paper is to develop a multilevel optimization model by mathematical programming that can be used for allocating the annual available limited water and land at various levels, maximizing the annual net benefit.

This paper describes a methodology that aims at defining optimization criteria for deficit irrigation of a farm. The methodology uses, as input, observed or estimated data of different variables (rainfall, evapotranspiration, applied water, soil-crop-unit characteristics etc.). (Mannochi and Mecarlli,1994). In the first phase, the crop yield –applied water relationships were simulated by budget model. Budget is composed of a set of validated subroutines describing the various processes involved in water extraction by plant roots and water movement in the soil profile. The relative yield decline that is expected under specific levels of water stress at different moments in the growing period is estimated by integrating the FAO ky approach in the soil water balance model Budget. (Raes *et al*. 2005).

2.MATRERIALS AND METHODS

2.1. Study Area

The Arasbaran district examined in the present study is located in the north-western of Iran at latitude 39o,13' N and longitude 47o,20' E. The existing command area under the Arasbaran reservoir is about 1500ha and falls under semiarid tropical region with an average annual rainfall of 278 mm. This areas are Irrigated by Fixed sprinkler system that interval of latrals are 25*25m. The soils in the area are dark brown to deep clay loam texture.

2.2. Crop yield and irrigation

Jensen (1968) proposed the following mathematical relationship between relative yield and the relative evapotranspiration:

$$\left(\frac{Y_a}{Y_m}\right) = \prod_{i=1}^{N} \left(\frac{ET_{act_i}}{ET_{crop_i}}\right)^{\lambda_i} \qquad (1)$$

where Y_a is the actual harvested yield, Y_m is the maximum crop yield under given management conditions that can be obtained when water is not limiting, N is the number of growth stages, $ETact_i$ is the actual evapotranspiration and $ETcrop_i$ is evapotranspiration for non-limiting water conditions during the ith stage of growth and li is the sensitivity index of crop to water stress during the ith stage of growth. The Jensen's model has two advantages. The first advantage is that it integrates the effect of all the water stress throughout the growing season. The second advantage is that the model can be used at time steps less than a growth stage. Sensitivity indexes of Jensen's model are related to yield response factors by the following polynomial function (Kipkorir *et al*. 2002):

$$\lambda = 0.2757ky^3 - 0.1351ky^2 + 0.8761ky - 0.0187$$

$R^2 = 0.99$ (2)

For any given Ky value, the corresponding l value can be derived with the help of Eqn(3) for any given growth stage. The sensitivity indexes for shorter time period less than growth stage can be subsequently determined using the procedure presented by Tsakiris (1982). The effect of water stress on the relative yield during a short time period is finally derived by means of the Jensen model. In the soil water balance model BUDEGET relative yield decline that is expected under specific levels of water stress at different moments in the growing period is estimated by integrating the FAO ky approach.

2.3. Simulations

Expected yields of crop patterns (wheat and barley) under different levels of water stress in Arasbaran climatic regions were simulated with the help of Budget for ten years (2000-2009). The climatic input consists of monthly reference evapotranspiration (ET_0) as determined by means of the FAO Penman-Manteith method (Allen *et al*.1998) and monthly rainfall as abserved in the nearby weather station of Arasbaran (Aslandoz station). The lengh of the growth stages, crop coefficients(Kc), rooting depths (Zr), soil water depletion factors for no stress(P), length of the sensitivity stages and yield response factors (Ky) were derived from indicative values presented by Allen *et al*. (1998) and Doorenbos and Kassam(1979). Simulations started at the sowing date by considering the soil water content in each year for each crop. The economic values necessary for calculate net benefits were supplied by recorded date for east Azarbayjan agriculture organization and agricultural center of Moghan (north of Iran). An application efficiency(EFI) of 70% was assumed as appropriate for sprinkler irrigation in the area under examination. The net benefits of suggested crops for years 2000 to 2009, calculated and are shown in table 1 to 4.

Table 1. Calculated net benefits of wheat for years 2000 to 2009.

Year	Net benefit (Ris/ha)	Sale price of yield (Ris/ton)	Fixed cost (Ris/ha)	Cost of production (Ris/ton)
2000	2,784,575	650,000	1,287,500	21,560
2001	3,266,507	800,000	1,565,501	25,200
2002	4,174,749	950,000	1,805,000	28,400
2003	6,468,980	1,200,000	2,167,000	42,000
2004	6,368,480	1,400,000	2,589,000	54,700
2005	7,070,107	1,600,000	2,887,200	67,600
2006	5,853,849	1,800,000	3,167,000	80,100
2007	9,422,943	2,100,000	3,498,000	94,500
2008	11,084,743	2,500,000	3,585,000	112,000
2009	11,903,320	3,000,000	3,640,000	128,500

Table 2. Calculated net benefits of barley for years 2000 to 2009.

Year	Net benefit (Ris/ha)	Sale price of yield (Ris/ton)	Fixed cost (Ris/ha)	Cost of production (Ris/ton)
2000	2,333,683	550,000	1,043,730	20,400
2001	2,889,445	750,000	1,269,155	70,500
2002	3,278,007	850,000	1,419,330	81,500
2003	5,366,920	1,150,000	911,520	92,300
2004	6,572,171	1,450,000	1,721,515	122,500
2005	9,037,574	1,700,000	2,107,140	125,000
2006	6,821,186	1,850,000	1,066,020	162,000
2007	12,700,809	2,300,000	2,821,160	154,000
2008	22,810,270	3,700,000	2,811,210	143,000
2009	10,384,729	2,800,000	2,827,420	158,000

Table 3. Calculated net benefits of cotton for years 2000 to 2009**.**

Year	Net benefit (Ris/ha)	Sale price of yield (Ris/ton)	Fixed cost (Ris/ha)	Cost of production (Ris/ton)
2000	2,715,962	2,100,000	2,258,010	238,001
2001	3,001,808	2,500,000	2,588,760	275,920
2002	4,916,202	2,950,000	2,295,510	279,446
2003	5,705,995	3,300,000	3,436,670	374,523
2004	5,493,880	3,640,000	2,499,080	465,701
2005	6,637,665	4,300,000	3,158,270	479,284
2006	9,727,743	5,550,000	1,822,670	522,280
2007	12,790,712	6,500,000	2,498,160	589,059
2008	12,649,497	7,400,000	2,515,430	820,069
2009	16,939,524	9,800,000	2,713,400	931,100

Table 4. Calculated net benefits of alfalfa for years 2000 to 2009**.**

Year	Net benefit (Ris/ha)	Sale price of yield (Ris/ton)	Fixed cost (Ris/ha)	Cost of production (Ris/ton)
2000	7,153,553	1,200,000	1,232,68	60,700
2001	6,802,239	1,500,000	1,447,280	126,500
2002	8,267,909	1,750,000	1,675,400	138,000
2003	13,864,739	2,050,000	2,137,500	183,400
2004	11,492,634	2,400,000	1,935,700	239,500
2005	16,270,244	2,950,000	2,437,600	258,600
2006	7,521,544	3,300,000	1,287,400	355,500
2007	20,364,821	3,700,000	3,155,500	387,500
2008	19,375,750	4,100,000	3,056,000	402,300
2009	10,120,185	4,300,000	3,078,450	488,500

2.4. Objective Functions and Optimization

The profit attainable are represented by the fllowing function:

$$Z = (NB_w * A_w) + (NB_b * A_b) + (NB_a * A_a) + (NB_c * A_c)$$

Where NBw= unit-profit per hectare for wheat, NBb= unit-profit per hectare for barley NBc= unit-profit per hectare for cotton and NBa= unit-profit per hectare for alfalfa. The mathematical programming envisages an objective function which maximizes Z, subject to the following constraints:

$$A_w + A_b + A_c + A_a = 1200ha \quad (3a$$

$$V_w * A_w + V_b * A_b + V_c * A_c + V_a * A_a = NIV \quad (3b)$$

$$0 \le Area_w \le 600ha \quad (3c)$$

$$100 \le Area_a \le 1200ha \quad (3d)$$

$$100 \le Area_b \le 1200ha \quad (3e)$$

Where NIV = net available irrigation volume for the total area of 1200 ha(3a) (NIV=8,10,12,14,16 MCM are examined in this research). Constraint (3b) states that wheat, barley, cotton and alfalfa can be irrigated by NIV and rate of NIV are obtained from part of selin river flows that can deliver to Arasbaran dam in each year (as can be seen in reality) and constraint (3c) imposes one agronomic condition: since, in a correct crop rotation, wheat cannot be followed by wheat(because there would be a drastic reduction in productivity), leading to the condition that the maximum area covered by this crop should be lower than or equal to 50% of the total area. Such a restriction does not exist for the other three crops under consideration. Constraint (3d), (3e) states that barley and alfalfa must be cultivated at least 100 ha to bestial needs.

System analysis using mathematical model provides a suitable methodology to analyze various aspect of water resource system planning. Linear Programming is used for this study.

3.RESULTS AND DISCUSSION

Linear programming was analyzed by solver at excel program to optimization of objective function(Z).

the analysis of historical data represents a possible means to obtain suggestion for the future. As can be seen, the optimum crop pattern shows variation from year to year, caused by the stochastic component which is linked to the climate. There are some years in which, it is possible to great area to cotton (with a same net irrigation supply). There are other years in which cotton is completely excluded from the crop pattern and the whole irrigation supply is used for other crops. There are yet other years in which wheat is cultivated at least. The annual net benefit attainable is obviously affected by the variability of the crop pattern and shows great variation strictly correlated to the chosen crops.

Staying within the realms of limited water availability, it seems evident that the impossibility of forecasting the weather at the beginning of the agricultural year necessitate a statistical synthesis of various crop patterns. This would allow a planner to correctly manage the means of production (mechanization, enlargement of the irrigation system, etc.) and ensure stable profit.

It should be stressed that, the maximum profit is not attained by cultivating just a few hectares of one crop and supplying irrigation on requirement, but rather, by cultivating a larger area in conditions of deficit irrigation.

Fig 1. shows comparison between annual net benefits as function of net available volume(NIV) in case of optimal variable pattern. [NIV=8 to 16MCM]. As is easily foreseen, the annual profits attainable with a variable NIV is increasing when the NIV is increasing. In some years as in 2003 and 2008 profits can at the maximum be equal to, it is because the rainfall in this years is high rate and effect of irrigation water is equal, But in the optimal crop pattern, unit net benefit attainable from the use of every unit of irrigation water is increasing when the available water(NIV) is decreasing.

4.CONCLUSION

The following conclusions can be drawn for the study area based on the results obtained from the model:

1. results reveal that with modeling of variable water resources there is a substantial increase in net annual benefit by optimal crop patterns. Especially when the available water is low.

2. the optimum crop pattern shows variation from year to year, caused by the stochastic component which is linked to the climate. There are some years in which, it is possible to great area to cotton (with a same net irrigation supply). There are other years in which cotton is completely excluded from the crop pattern and the whole irrigation supply is used for other crops. There are yet other years in which wheat is cultivated at least.

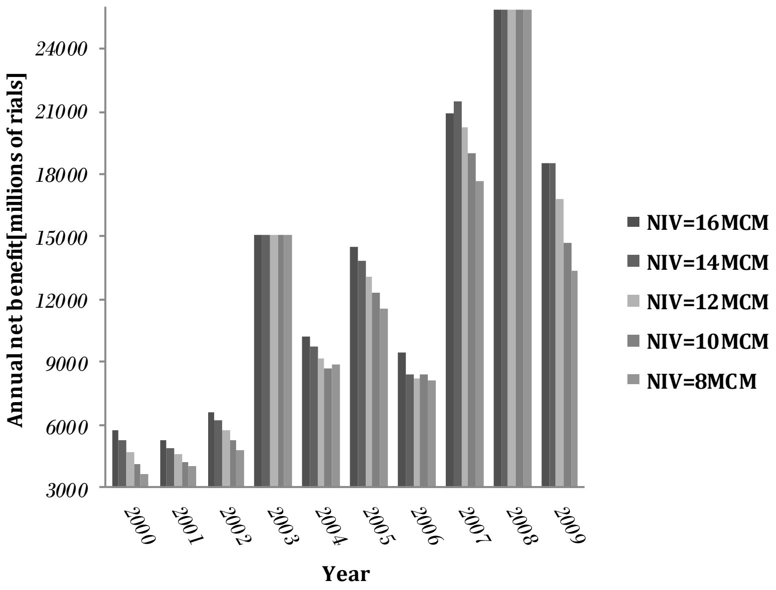

Figure 1. Comparison between annual net benefits as function of net available volume(NIV)

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