A bi-objective optimization approach to a municipal solid waste management system

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Abstract

Municipal solid waste management is one of the most important elements of urban management, especially for those metropolises dealing with increasing waste generation. To support effective municipal solid waste management systems, the economic factor is considered as one of the most critical criteria to be taken into account. While minimizing the opening cost of the waste management system's facilities is considered as one of the main factors in view of the government authorities such as municipalities, minimizing the transportation cost is the main interest of the sectors responsible for handling and transferring wastes (mostly contracted private sectors). Taking these two factors into account, this study proposes a bi-objective mixed-integer programming model for a municipal solid waste management system to optimize the total cost. Utilizing the lexicographic optimization method, the proposed model is successfully implemented in the general algebraic modeling system optimization software and obtained more efficient results than the common approaches of a single objective modeling.

Keywords: Municipal Solid Waste (MSW); Mixed-Integer Programming (MIP); Location-Routing Problem (LRP); Lexicographic optimization

1. Introduction

To support decision-making associated with Municipal Solid Waste (MSW) management, Operation Research (OR) techniques are widely utilized to construct mathematical models for MSW management systems (Ghiani *et al*., 2014). Among many OR techniques, Mixed-Integer Programming (MIP) models have been applied to locating a system component (e.g. transfer stations) or determining the flow of wastes within the system with respect to minimizing the total MSW management cost (Ghiani *et al*., 2014, Tan *et al*., 2014). More detailed studies on the applications of OR to MSW management was reviewed by Ghiani *et al*., (2014).

Applications of MIP in MSW management have been focused on either a location selection problem or a waste flow optimization in the system (Badran and El-Haggar, 2006, Eiselt, 2007, Santibañez-Aguilar *et al*., 2013, Nga *et al*., 2013, Tan *et al*., 2014), and addressing both locating and routing simultaneously has been rarely studied in the context of MSW management. Erkut *et al*. (2008) proposed a multi-objective MIP model to design a MSW management system in the frame of a Location-Routing Problem (LRP). They utilized the developed model to compare the scenarios of planning regional and prefectural MSW management in central Macedonia. The employed solution involved a lexicographic optimization method to solve the model and obtain non-dominated solutions. Further, Asefi *et al*. (2015) proposed a MIP model to minimize the total cost of a MSW management system while formulating a single objective model and considering some simplifying assumptions such as a single flow of MSW from generation to the intermediate facilities.

The present study aims to address a MSW management system where the system's all components (i.e. transfer stations, treatment centers, recycling facilities and disposal units) are considered simultaneously. Multiple types of MSW including recyclables, Household Hazardous Waste (HHW) and Waste Electrical and Electronic Equipment (WEEE) are considered while the limitations of waste-to-treatment processes are also taken into account. A multi-objective MIP model is proposed for the addressed problem. The model is formulated using the General Algebraic Modeling System (GAMS) optimization software and the lexicographic optimization method is applied to solve the model.

2. MSW management system

2.1. Conceptual model

The considered MSW system is schematically shown in Figure 1. The model considers all the system components and addresses routing wastes and residues among the generation sources and the components.

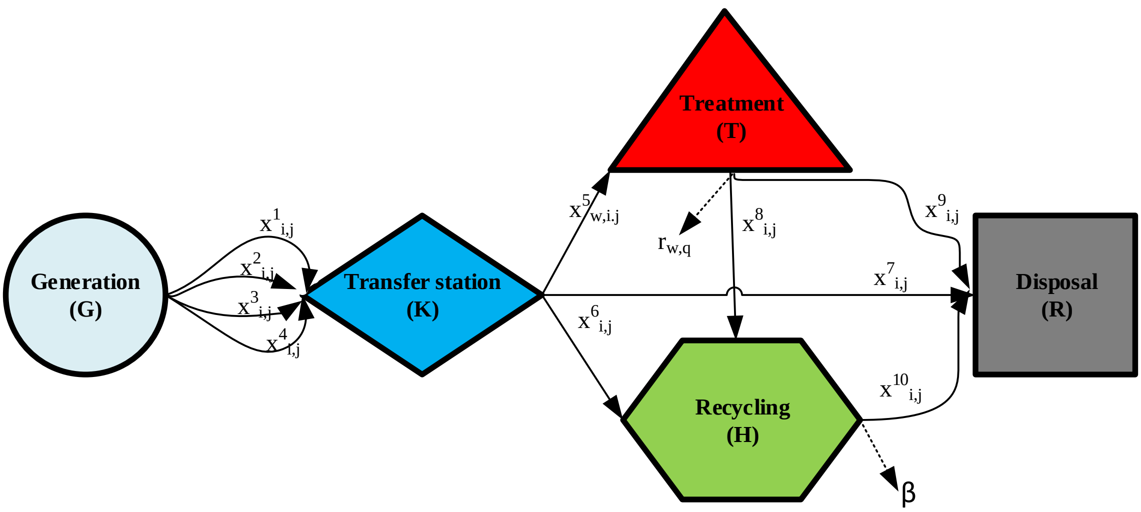

Figure 1. MSW management system

2.2. Mathematical model

Equations (1) and (2) are formulated to determine the objective functions. Minimizing the establishment cost and minimizing the transportation cost are formulated by Equations (1) and (2), respectively. Equations (3)-(16) determine the flow balance in the system. That is, using mass input-output relation constraints, no processing site may keep the waste. The total input waste to the system has to be processed. Equations (17)-(20) ensure that the total load of a system component is not greater than the component capacity. Equations (21)-(24) state the minimum amount requirements. That is, a system component is opened only if the minimum amount of waste processed by the facility is available. Equation (25) ensures compatibility between the waste type and treatment technology. Finally, Equations (26) and (27) formulate non-negativity constraints for flow variables and binary variables of location selection.

$$Minimize\ f_1(x) = \sum_{i\in K} z_i^K\ a_i^K + \sum_{i\in T}\sum_{q\in Q} z_{q,i}^T a_{q,i}^T + \sum_{i\in H} z_i^H\ a_i^H + \sum_{i\in R} z_i^R\ a_i^R \quad (1)$$

$$Minimize\ f_2(x) = \sum_{i\in G}\sum_{j\in K} c_{i,j}^1 x_{i,j}^1 + \sum_{i\in G}\sum_{j\in K} c_{i,j}^2 x_{i,j}^2 + \sum_{i\in G}\sum_{j\in K} c_{i,j}^3 x_{i,j}^3 + \sum_{i\in G}\sum_{j\in K} c_{i,j}^4 x_{i,j}^4 + \sum_{i\in K}\sum_{j\in T}\sum_{w\in W} c_{i,j}^5 x_{w,i,j}^5 + + \sum_{i\in K}\sum_{j\in H} c_{i,j}^6\ x_{i,j}^6 + \sum_{i\in K}\sum_{j\in R} c_{i,j}^7\ x_{i,j}^7 + \sum_{i\in T}\sum_{j\in H} c_{i,j}^8\ x_{i,j}^8 + \sum_{i\in T}\sum_{j\in R} c_{i,j}^9\ x_{i,j}^9 + + \sum_{i\in H}\sum_{j\in D'} c_{i,j}^{10}\ x_{i,j}^{10} \quad (2)$$

subject to

$$g_i^1 = \sum_{j\in K} x_{i,j}^1 \qquad \forall i \in G \quad (3)$$

$$g_i^2 = \sum_{j\in K} x_{i,j}^2 \qquad \forall i \in G \quad (4)$$

$$g_i^3 = \sum_{j\in K} x_{i,j}^3 \qquad \forall i \in G \quad (5)$$

$$g_i^4 = \sum_{j\in K} x_{i,j}^4 \qquad \forall i \in G \quad (6)$$

$$\sum_{i\in G} x^1{}_{i,j} + \sum_{i\in G} x^2{}_{i,j} + \sum_{i\in G} x^3{}_{i,j} + \sum_{i\in G} x^4{}_{i,j} = N_j^K \quad \forall j \in K \quad (7)$$

$$\sum_{j\in G} x^3{}_{j,i}\, O_w + \sum_{j\in G} x^4{}_{j,i}\, O'_w = \sum_{j\in T} x^5{}_{w,i,j} \quad \forall w \in W, \forall i \in K \quad (8)$$

$$\sum_{j\in G} x^1{}_{j,i} + \sum_{j\in G} x^2{}_{j,i} + \sum_{j\in G} x^3{}_{j,i}\, O'' = \sum_{j\in H} x^6{}_{i,j} \quad \forall i \in K \quad (9)$$

$$\sum_{j\in G} x^3{}_{j,i}\left(1 - \left(\sum_{w\in W} O_w + O''\right)\right) = \sum_{j\in R} x^7{}_{i,j} \quad \forall i \in K \quad (10)$$

$$\sum_{i\in K} x^5{}_{w,i,j} = \sum_{q\in Q} N_{w,q,j}^T \quad \forall w \in W, \forall j \in T \quad (11)$$

$$\sum_{q\in Q}\sum_{w\in W} N_{w,q,i}^T\left(1 - r_{w,q}\right) r'_{w,q} = \sum_{j\in H} x_{i,j}^8 \quad \forall i \in T \quad (12)$$

$$\sum_{i\in K} x^6{}_{i,j} + \sum_{i\in T} x^8{}_{i,j} = N_j^H \quad \forall j \in H \quad (13)$$

$$\sum_{w\in W}\sum_{q\in Q} N_{w,q,i}^T\left(1 - r_{w,q}\right)\left(1 - r'_{w,q}\right) = \sum_{j\in R} x_{i,j}^9 \quad \forall i \in T \quad (14)$$

$$N_i^H(1 - \beta_i) = \sum_{j\in R} x_{i,j}^{10} \quad \forall i \in H \quad (15)$$

$$\sum_{i\in K} x_{i,j}^7 + \sum_{i\in T} x_{i,j}^9 + \sum_{i\in H} x_{i,j}^{10} = N_j^R \quad \forall j \in R \quad (16)$$

$$N_i^K \le P_i^K\ a_i^K \quad \forall i \in K \quad (17)$$

$$\sum_{w\in W} N_{w,q,i}^T \le P_{q,i}^T\ a_{q,i}^T \quad \forall q \in Q, \forall i \in T \quad (18)$$

$$N_i^H \le P_i^H\ a_i^H \quad \forall i \in H \ (19)$$

$$N_i^R \le P_i^R\ a_i^R \quad \forall i \in R \ (20)$$

$$N_i^K \ge P_i'^K\ a_i^K \quad \forall i \in K \ (21)$$

$$\sum_{w\in W} N_{w,q,i}^T \ge P_{q,i}'^T\ a_{q,i}^T \quad \forall q \in Q, \forall i \in T \quad (22)$$

$$N_i^H \ge P'^H_i\ a_i^H \quad \forall i \in H \quad (23)$$

$$N_i^R \ge P'^R_i\ a_i^R \quad \forall i \in R \quad (24)$$

$$N_{w,q,i}^T \le P_{q,i}^T\, y_{w,q} \quad \forall w \in W, \forall t \in T, \forall i \in M \quad (25)$$

$$\left(x_{i,j}^1, x_{i,j}^2, x_{i,j}^3, x_{i,j}^4, x_{w,i,j}^5, , x_{i,j}^6, x_{i,j}^7, x_{i,j}^8, x_{i,j}^9, x_{i,j}^{10}, N_i^K, N_{w,q,i}^T, N_i^H, N_i^R\right) \in \{\mathbb{R}^{14}\}^+ \quad (26)$$

$$\left(a_i^K, a_{q,i}^T, a_i^H, a_i^R\right) \in \{0,1\}^4 \quad (27)$$

3. Implementation

3.1. Lexicographic optimization

The lexicographic method considers that the objectives can be prioritized in the order of importance. Here, the establishment cost (f_1) has greater influence on the total cost compared with the transportation cost (f_2). Accordingly, we assume greater importance for f_1 than f_2 and solve the proposed MIP model in the form shown in Problem (I). The lexicographic optimization method consists of solving a sequence of single objective optimization problems where each problem optimizes one of the objectives separately.

$$\begin{aligned} &\min f_1(x) \\ &s.t. \qquad\qquad\qquad (I) \\ &f_2(x) \le y_2^* \end{aligned}$$

$x \in X$,

where y_2^* is the result of the objective f_2 obtained by solving the Problem (II) with respect to this objective solely, and X denotes the feasible area.

$$\min f_2(x) \quad s.t. \quad x \in X , \qquad \text{(II)}$$

3.2. Numerical results

The single objective and the proposed bi-objective solution approaches are evaluated with respect to the resulted total cost (i.e. the total cost of establishment and transportation). To justify the accuracy of the proposed model and efficiency of the applied method, having the data and parameters presented in (Asefi *et al.*, 2015), the two solution approaches are implemented in GAMS as described below.

- ***Single objective solution approach***: the model is formulated as shown in Problem (III) where f_3 denotes the objective of total cost which is summation of the total cost of establishment and the total transportation cost. Let y_{Single}^* denotes the resulted objective value of the approach.

$$\min f_3(x) \quad s.t. \quad x \in X , \qquad \text{(III)}$$

- ***Bi-objective solution approach using lexicographic method***: the proposed MIP model is formulated as shown in Problem (I) where the objective of minimizing the transportation cost (f_2) is bound as a constraint in the model. Let y_1^* denote the resulted objective value of the model, the total cost of this approach is then denoted by $y_{bi-objective}^*$ where calculated as $(y_1^* + y_2^*)$.

The obtained results by the solution approaches are presented in Table 1. As can be seen in Table 1, the total cost obtained by the bi-objective lexicographic method ($y_{bi-objective}^* = 9.22\text{E} + 08$) is lower than the total coast obtained when the problem is solved in the single objective form ($y_{Single}^* = 9.23\text{E} + 08$). The more detailed investigation on the implemented case showed that the resulted improvement in the total cost is decreased by 7.46% in the transportation cost of the system.

Table 1. Numerical results

Solution approach	**Establishment cost**	**Transportation cost**	**Total cost**
Single objective	_	_	9.23E+08
Bi-objective Lexicographic	9.10E+08	1.22E+07	9.22E+08

4. Concluding remarks

To effectively tackle the addressed problem of an integrated municipal solid waste location-routing problem , the present study proposed a bi-objective MIP model to minimize the total cost of the system consisting the opening cost of the facilities and the transportation cost within the entire system. The lexicographic optimization method is successfully applied to the problem. The result shows that, while binding one of the objectives as a constraint, the total cost is minimized and we were able to obtain more efficient solutions than the common approaches of formulating the total cost in a single objective.

The present study could still be enriched by further studies. One possibility could be taking consideration of different technologies for recycling centers. In terms of a solution approach, proposing efficient heuristic methods could assist the decision-makers when dealing with the problem in large scales.

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Nomenclature

$N = (V, A)$ transportation network of nodes V and arcs A

$G = \{1, \dots, g\}$ set of waste generation nodes, $G \in V$

$K = \{1, \dots, k\}$ set of potential transfer station nodes, $K \in V$

$T = \{1, \dots, t\}$ set of potential treatment nodes, $M \in V$

$H = \{1, \dots, h\}$ set of potential recycling/recovering nodes, $H \in V$

$R = \{1, \dots, r\}$ set of potential non-hazardous disposal nodes, $L \in V$

$W = \{1, \dots, w\}$ set of hazardous waste types

$Q = \{1, \dots, q\}$ set of treatment technologies

g_i^1 amount of dry recyclable waste generated at generation node $i \in G$

g_i^2 amount of organic recyclable waste generated at generation node $i \in G$

g_i^3 amount of mixed-waste garbage generated at generation node $i \in G$

g_i^4 amount of WEEE & HHW generated at generation node $i \in G$

${c^1}_{i,j}$ transportation cost per unit of dry recyclable waste on link $(i,j) \in A,\ i \in G, j \in K$

${c^2}_{i,j}$ transportation cost per unit of organic recyclable waste on link $(i,j) \in A, i \in G, j \in K$

${c^3}_{i,j}$ transportation cost per unit of mixed-waste garbage on link $(i,j) \in A, i \in G, j \in K$

${c^4}_{i,j}$ transportation cost per unit of WEEE & HHW waste on link $(i,j) \in A, i \in G, j \in K$

$c_{w,i,j}^5$transportation cost per unit of hazardous waste sorted as type $w \in W$ on link $(i,j) \in A, i \in K, j \in T$

$c_{i,j}^6$transportation cost per unit of recyclable waste on link $(i,j) \in A, i \in K, j \in H$

${c^7}_{i,j}$ transportation cost per unit of garbage residues on link $(i,j) \in A, i \in K, j \in R$

$c_{i,j}^8$transportation cost per unit of recyclable residues sorted on link $(i,j) \in A, i \in T, j \in H$

$c_{i,j}^9$transportation cost per unit of residues on link $(i,j) \in A, i \in T, j \in R$

${c^{10}}_{i,j}$ transportation cost per unit of residues on link $(i,j) \in A, i \in H, j \in R$

z_i^K fixed cost of opening a transfer station unit at node $i \in K$

$z_{q,i}^T$ fixed cost of opening a treatment technology $q \in Q$ at node $i \in T$

z_i^H fixed cost of opening a recycling\recovering unit at node $i \in H$

z_i^R fixed cost of opening a disposal unit at node $i \in R$

$r_{w,q}$ proportion of mass reduction of treated waste of type $w \in W$ under technology $q \in Q$

${r'}_{w,q}$ proportion of recycling of treated waste type $w \in W$ after undergoing treatment technology $q \in Q$

β proportion of total waste recycled at node $i \in H$

O_w proportion of mixed-waste garbage which is sorted as hazardous type $w \in W$

${O'}_w$ proportion of WEEE & HHW waste which is sorted as hazardous type $w \in W$

o'' proportion of mixed-waste garbage which is sorted as recyclable

P_i^K capacity of transfer station at node $i \in K$

$P_{q,i}^T$ capacity of treatment technology $q \in Q$ at node $i \in T$

P_i^H capacity of recycling\recovering center at node $i \in H$

P_i^R capacity of disposal unit at node $i \in R$

${P'}_i^K$ the minimum amount of waste required to open a transfer station unit at node $i \in K$

${P'}_{q,i}^T$ the minimum amount of waste and residues required to open a treatment unit under technology $q \in Q$ at node $i \in T$

${P'}_{r,i}^H$ the minimum amount of waste and residues required to open a recycling\recovering centerat node $i \in H$

${P'}_i^R$ the minimum amount of garbage and residues required to open a disposal unit at node $i \in L$

${P'}_i^{L'}$ the minimum amount of hazardous residues required to open a hazardous disposal unit at node $i \in R$

$y_{w,q}$ 1 if hazardous waste\residue type $w \in W$ is compatible with technology $q \in Q$; or 0 otherwise

${x^1}_{i,j}$ amount of dry recyclable waste transported through link $(i,j) \in A,\ i \in G, j \in K$

${x^2}_{i,j}$ amount of organic recyclable waste transported through link $(i,j) \in A, i \in G, j \in K$

${x^3}_{i,j}$ amount of mixed-waste garbage transported through link $(i,j) \in A, i \in G, j \in K$

${x^4}_{i,j}$ amount of WEEE & HHW waste transported through link $(i,j) \in A, i \in G, j \in K$

$x_{w,i,j}^5$ amount of waste sorted as hazardous type $w \in W$ transported through link $(i,j) \in A, i \in K, j \in T$

$x_{i,j}^6$ amount of waste sorted as recyclable transported through link $(i,j) \in A, i \in K, j \in H$

${x^7}_{i,j}$ amount of waste residues transported through link $(i,j) \in A, i \in K, j \in R$

$x_{i,j}^8$ amount of recyclable residues of treatment centers transported through link $(i,j) \in A, i \in T, j \in H$

$x_{i,j}^9$amount of residues of treatment centers transported through link $(i,j) \in A, i \in T, j \in R$

${x^{10}}_{i,j}$ amount of residues transported through link $(i,j) \in A, i \in H, j \in R$

N_i^K amount of waste sorted and transferred at node $i \in K$

$N_{w,q,i}^T$ amount of waste type $w \in W$ treated at node $i \in T$ under technology $q \in Q$

N_i^H amount of waste recycled at node $i \in H$

N_i^R amount of residue disposed at node $i \in R$

a_i^K 1 if transfer station unit is opened at node $i \in K$; or 0 otherwise.

$a_{t,i}^T$ 1 if treatment unit of technology unit $q \in Q$ is opened at node $i \in T$; or 0 otherwise

a_i^H 1 if recycling\recovering unit is opened at node $i \in H$; or 0 otherwise

a_i^R 1 if disposal unit is opened at node $i \in R$; or 0 otherwis